CONCLUSIONS

1. It appears more appropriate to assume that the bubble shape is represented by a spherical segment characterized by the base radius R_1 and the vertical height H.
2. A general expression has been developed for initial microlayer thickness.
3. The bubble growth curves obtained on the basis of the assumption of "infinitely thick microlayer", i.e. $k_s \rho_s Cp_s = k_L \rho_L Cp_L$ are in satisfactory agreement with experimental results.

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EFFECT OF JAKOB NUMBER ON FORCES CONTROLLING BUBBLE DEPARTURE IN NUCLEATE POOL BOILING

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NOMENCLATURE

C_d, C_{do}, drag coefficients;
C_{pL}, specific heat of liquid [J/kg-K];
D, bubble diameter [m];
D_c, diameter of bubble neck in contact with heating surface at departure [m];
D_d, bubble diameter at departure [m];
F, force normal to heating surface [N];
g, acceleration due to gravity [m/s^2];
h, average heat-transfer coefficient [W/m^2-K];
h_{fg}, latent heat of vaporization of bulk liquid [J/kg];
k, thermal conductivity of liquid [W/m-K];
N_{Ja}, Jakob number ($C_{pL}\rho_L \Delta T/h_{fg}\rho_V$);
$\dot{q}$, heat flux [W/m^2];
t, time [s];
T_w, wall temperature [°K];
T_{sat}, saturation temperature [°K];
T_∞, bulk liquid temperature [°K];
ΔT, wall superheat [°K];
V_g, velocity of center of bubble, $\frac{1}{2}\frac{dD}{dt}$ [m/s];
V_n, normal velocity of bubble front, dD/dt [m/s];
V_t, terminal velocity [m/s];
δ, thickness of superheated liquid layer [m];
ρ_L, liquid density [kg/m^3];
ρ_V, vapor density [kg/m^3];
σ, coefficient of surface tension [N/m];
θ, contact angle [rad].

INTRODUCTION

It is well known that at low values of Jakob number the bubble growth rates and departure diameters are small and bubble departure is controlled primarily by the surface tension force, the inertia (drag and liquid inertia) forces being relatively small. At high values of Jakob number, on the other hand, the growth rates and departure diameters are large and inertia forces control bubble departure. The object of this paper is to obtain quantitatively the Jakob number ranges over which the surface tension force and inertia forces control, respectively, the process of bubble departure.

THEORETICAL ANALYSIS

Assumptions

The following assumptions have been made:

1. At the instant of departure the bubble is spherical in shape and it is attached to the heating surface by a short neck of diameter D_c and having a contact angle $\theta \approx \pi/2$.

The neck diameter is the arithmetic mean of the two values obtained from the expression [1]

$$2\times\frac{\delta\,(T_w-T_{sat})}{3\,(T_w-T_\infty)}\left\{1+\left[1-\frac{12(T_w-T_\infty)T_{sat}\,\sigma}{(T_w-T_{sat})^2\delta\rho_V\,h_{fg}\,J}\right]^{1/2}\right\} \quad (1)$$

where δ denotes the thickness of the superheated liquid layer adjacent to the surface.

2. The value of δ may be estimated by [2]

$$\delta = 1{\cdot}65k/h \quad (2)$$

where $h = \dot{q}/\Delta T$ represents the average heat-transfer coefficient.

Forces acting on bubble

The various forces acting on a bubble are:

(i) Buoyancy force F_B

$$\frac{\pi D^3}{6}(\rho_L-\rho_V)g \quad (3)$$

(ii) Surface tension force F_{ST}

$$= \pi D_c\,\sigma\sin\theta \quad (4)$$

(iii) Liquid inertia force F_{LI}

$$= \frac{1}{2}\frac{\pi D^3}{6}\rho_L\frac{\mathrm{d}\,V_n}{\mathrm{d}t}$$

[2] (5)

$$= \frac{1}{2}\frac{\pi D^3}{6}\rho_L\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)$$

(iv) Bubble inertia force F_{BI}

$$= \frac{\pi D^3}{6}\rho_V\frac{\mathrm{d}\,V_g}{\mathrm{d}t}+\frac{\pi}{6}\rho_V\,V_g\frac{\mathrm{d}D^3}{\mathrm{d}t}$$

[2] (6)

$$= \frac{\pi D^3}{6}\rho_V\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\frac{\mathrm{d}D}{\mathrm{d}t}\right)+\frac{\pi}{6}\rho_V\,V_g\frac{\mathrm{d}D^3}{\mathrm{d}t}$$

(v) Viscous drag force F_{CD}

$$= C_d\,\rho_L\frac{\pi D^2}{4}\frac{V_n^2}{2}$$

$$= C_d\,\rho_L\frac{\pi D^2}{8}\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)^2 \quad (7)$$

where $C_d = C_{do}/(V_g/V_t)$.
C_{do} is given by [2]

$$C_{do} = 2{\cdot}0 \quad (8)$$

and

$$V_t = \sqrt{[\tfrac{4}{3}g(\rho_L-\rho_V)D/\rho_L\,C_{do}]}.$$

Force balance at departure

This may be expressed as

$$F_B = F_{ST}+F_{CD}+F_{LI}+F_{BI}. \quad (9)$$

Since F_{BI} is small compared to other forces, equation (9) may be closely approximated by

$$F_B = F_{ST}+(F_{CD}+F_{LI}). \quad (10)$$

Equation (10) states that the moment the bubble departs from the surface the buoyancy force is just balanced by the sum of surface tension and inertia (drag and liquid inertia) forces.

RESULTS

Using experimental bubble growth data of Cole *et al.* [3] and Akiyama *et al.* [4], the forces F_B, F_{ST} and $(F_{CD}+F_{LI})$ have been computed and are plotted as function of Jakob number in Fig. 1 and of experimental bubble departure diameter in Fig. 2. It appears from these plots that

$F_B \approx F_{ST}$ for $\log N_{Ja} < 1{\cdot}2$

or $\log(D_d\times 10^3) < 0{\cdot}20$ (11)

$F_B \approx F_{CD}+F_{LI}$ for $\log N_{Ja} > 2{\cdot}0$

or $\log(D_d\times 10^3) > 1{\cdot}0$. (12)

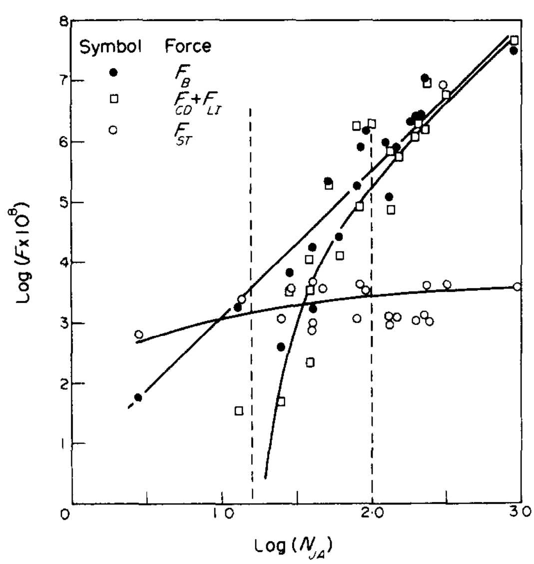

FIG. 1. Forces acting on bubble as function of Jakob number.

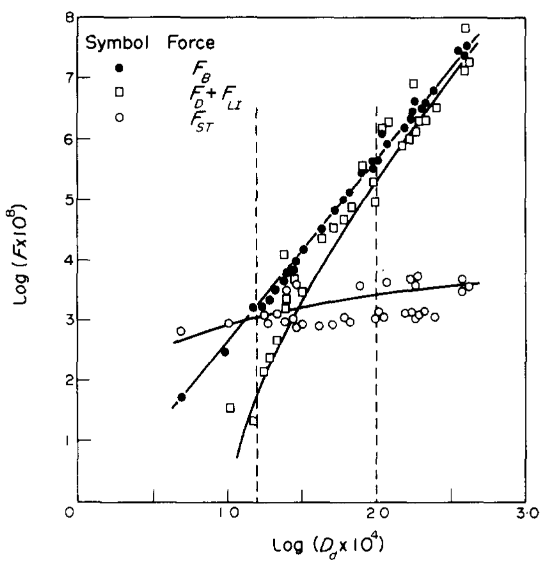

FIG. 2. Forces acting on bubble as function of departure diameter.

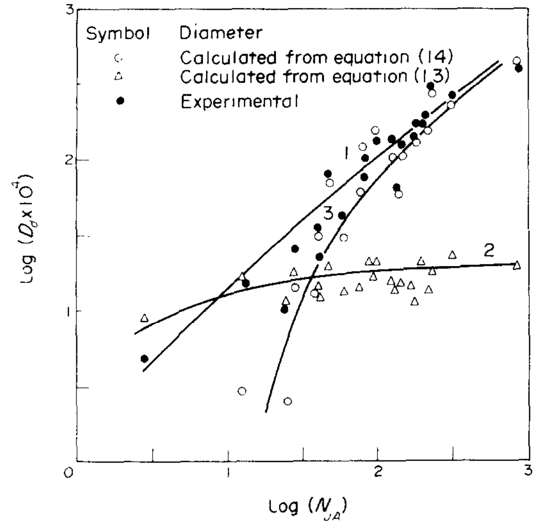

FIG. 3. Experimental and theoretical departure diameters.

It seems, therefore, that for departure diameters less than about 1·6 mm or for Jakob numbers less than about 16, the bubble departure is controlled by surface tension force while for $D_d > 10$ mm (approx.) or $N_{Ja} > 100$ (approx.) the inertia forces control bubble departure. For departure diameters between 1·6 and 10 mm (approx.) or N_{Ja} between 16 and 100 (approx.) the surface tension and inertia forces are of nearly equal importance.

Figure 3 shows D_d as function of N_{Ja}. Curve 1 is a plot of experimental departure diameters while curves 2 and 3 show respectively the values of D_d obtained from the following equations:

$$F_B = F_{ST} \tag{13}$$

$$F_B = F_{CD} + F_{LI}. \tag{14}$$

It is evident that values of D_d predicted by (14) are in reasonable agreement with experiment for $N_{Ja} > 100$ while for $N_{Ja} < 16$ the experimental values are closer to those predicted by equation (13).

CONCLUSION

Equations (13) and (14) predict satisfactorily the bubble departure diameter for $N_{Ja} < 16$ (approx.) or $N_{Ja} > 100$ (approx.), respectively. For intermediate values of N_{Ja} the departure diameter should be computed from equation (10).

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THERMAL CONVECTION IN A TILTED POROUS LAYER

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NOMENCLATURE

h, thickness of model;
ΔT, temperature difference between lower and upper plane;
g, acceleration of gravity;
x, y, z, dimensionless Cartesian coordinates;
k, m, dimensionless wave numbers in the x- and z-direction, respectively;
Ra, Rayleigh number.

Greek symbols

α, dimensionless overall wave number;
θ, dimensionless perturbation temperature;
φ, tilt angle with respect to the horizontal.

INTRODUCTION

THIS note is concerned with free, thermal convection in a porous layer being tilted an angle φ with respect to the horizontal. The layer is of infinite extent, and is bounded by two impermeable perfectly conducting planes separated by a distance h. The upper and lower planes are maintained at constant temperatures $-\Delta T/2$ and $\Delta T/2$, respectively. Both from a geophysical and technical point of view this type of flow is of considerable interest, and especially the horizontal layer problem is well described in the literature. Concerning a tilted porous layer, however, published works are not numerous. Most recently Bories and Combarnous [1] have studied this problem. Their main experimental results may be stated as follows. At small Rayleigh numbers